STUDENT NUMBER:

QUEEN'S UNIVERSITY, Smith Engineering
APSC 174 - All Sections - Introduction to Linear Algebra
Final Exam[1] – April 2025
Instructors: A. Ableson, M. Roth, L. Steverango, K. Zhang

HAND IN
answers recorded
on exam paper

Sample Multiple-Choice Problems

- Write your name and student number on this page before you begin.
- This examination is 3 HOURS in length.
- Answer all questions in the space provided. If you need more room, continue your answer on one of the blank pages at the back, **providing clear directions to the marker, e.g., Continued on Page 15**.
- Question 1 contains 16 multiple-choice problems. A Bubble Sheet for multiple-choice problems is provided at the end of this package. Please use the Bubble Sheet to record your responses. Note that answers written on the question pages (pages 2-7) will not be graded.
- Questions 2-5 are long answer questions. For full marks, you must show all your work and explain how you arrived at your answers. Correct answers with incorrect support may be given no credit.
- Only answers written in this booklet will be graded; all scrap paper from the test will be discarded.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Q 1	Q 2	Q 3	Q 4	Q 5	Total
16	10	7	10	5	48

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Question 1. Multiple-Choice Problems [1 mark each]

Each problem has several possible answers, labeled (A), (B), (C), and/or (D).

Choose the **most appropriate answer** and completely fill in the rectangle associated with your response for each problem on the attached **Bubble Sheet** (last page of this exam package).

Note the Bubble Sheet is not replaceable, so please follow these instructions carefully:

- mark only one answer per question;
- use a pencil (*not* pen) to allow erasing if you change your answer;
- fill in the bubbles completely;
- erase any mistakes completely;
- do not fold, tear, or write outside the bubble area.

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(1) Which of the following are linear combinations of $\mathbf{v}_1 = (0, -1, 1)$ and $\mathbf{v}_2 = (1, 2, 1)$?

(A) $(0, 0, 0)$
(B) $(2, 2, 4)$
(C) Both A and B
(D) Neither A nor B

(2) Are the vectors $\mathbf{v}_1 = (2, 0, -1)$, $\mathbf{v}_2 = (40, 0, 7)$, and $\mathbf{v}_3 = (-1, 100, 4)$ linearly dependent in $\mathbb{R}^3$?

(A) Yes
(B) No
(C) Not enough information to determine
(D) None of the above

(3) Which of the following is true?

(A) Every linearly independent set of seven vectors in $\mathbb{R}^7$ forms a basis for $\mathbb{R}^7$.
(B) Every set of seven vectors that spans $\mathbb{R}^7$ forms a basis for $\mathbb{R}^7$.
(C) Every set of vectors that spans $\mathbb{R}^7$ contains a basis for $\mathbb{R}^7$.
(D) All are true.

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(4) Consider the vector space $\mathbb{R}^2$ and $\mathbf{v} = (2, 0) \in \mathbb{R}^2$. Then the linear span of $\mathbf{v}$ is

(A) $\{\mathbf{v}\}$

(B) $\{(x, y) : \ x \in \mathbb{R}, \ y = 0\}$

(C) $\{(0, y) : \ y \in \mathbb{R}\}$

(D) $\mathbb{R}^2$

(5) If A is a 6×4 matrix, then the number of leading ones in the reduced row echelon form of A is at most

(A) 3

(B) 4

(C) 5

(D) 6

(6) To determine whether a set of n vectors from $\mathbb{R}^n$ is linearly independent, we can form a matrix A whose columns are the vectors in the set and then put A into its reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

(A) A row of all zeros

(B) A column of all zeros

(C) An identity matrix

(D) A row that has all zeros except in the last position

(7) A square matrix A is invertible if and only if

(A) $\lambda = 0$ is not an eigenvalue of A

(B) $\lambda = 0$ is an eigenvalue of A

(C) $\lambda = 1$ is not an eigenvalue of A

(D) $\lambda = 1$ is an eigenvalue of A

(8) If A and B are $n \times n$ real matrices, A is invertible, and $AB = \mathbf{0}$, then $B =$

(A) Zero matrix

(B) Identity matrix

(C) Invertible matrix

(D) The inverse of A

(9) If A is an $n \times n$ matrix, and columns of A are linearly dependent, then

(A) $\det(A) = 0$

(B) $\det(A) \neq 0$

(C) $\det(A) < 0$

(D) $\det(A) > 0$

(10) What is the determinant of the matrix $\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$?

(A) 2^4

(B) 2

(C) 1

(D) 0

(11) The characteristic polynomial of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ is

(A) $\lambda(\lambda - 2)$

(B) $(\lambda - 2)(\lambda + 1)$

(C) $\lambda(\lambda + 2)$

(D) $(\lambda - 1)(\lambda - 2)$

(12) Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. What is $A^{27}\mathbf{v}$?

(A) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ 2^{27} \end{bmatrix}$

(D) $\begin{bmatrix} a^{27} & b^{27} \\ c^{27} & d^{27} \end{bmatrix}$

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Questions (13) to (16) all refer to the following information. A linear transformation L has the standard matrix A shown below. Also shown is the reduced row echelon form (RREF) of A.

$$A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 3 & -9 & 4 \\ 0 & 2 & -6 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(13) Which of the following describes the input and output spaces for L?

(A) $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

(B) $L : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$

(C) $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$

(D) $L : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$

(14) What is $\dim(\text{Im}(L))$?

(A) $\dim(\text{Im}(L)) = 1$

(B) $\dim(\text{Im}(L)) = 2$

(C) $\dim(\text{Im}(L)) = 3$

(D) $\dim(\text{Im}(L)) = 4$

(15) What is $\dim(\text{Ker}(L))$?

(A) $\dim(\text{Ker}(L)) = 0$

(B) $\dim(\text{Ker}(L)) = 1$

(C) $\dim(\text{Ker}(L)) = 2$

(D) $\dim(\text{Ker}(L)) = 3$

(16) Which of the following vectors are in the kernel of L?

(A) $\begin{bmatrix} 0 \\ 6 \\ 2 \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ -2 \\ 6 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} -2 \\ 1 \\ -3 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 \\ 3 \\ 1 \\ 3 \end{bmatrix}$

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Note for students

These are a sample of possible questions; the actual exam questions may cover other material from the course.

Correct answers are:

1-(C)
2-(B)
3-(D)
4-(B)
5-(B)

6-(C)
7-(A)
8-(A)
9-(A)
10-(D)

11-(A)
12-(A)
13-(B)
14-(B)
15-(C)

16-(A)

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Name: _____ **Student ID Number:** _____

Instructions:

Please completely fill in the rectangle associated with your response. Example: ■ B C D E

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