STUDENT NUMBER: ______

QUEEN'S UNIVERSITY
FACULTY OF ENGINEERING AND APPLIED SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
APSC 174 FINAL EXAMINATION - APRIL 2018
INSTRUCTORS: MANSOURI, GHARESIFARD, YUI

INSTRUCTIONS

- This examination is **3 hours** in length and consists of **6 questions**.
- **READ THE QUESTIONS CAREFULLY!**
- Answer **all questions**, writing **clearly** in the space provided.
- If you need more room, there are blank pages at the end of the test. **If you use these pages, you must provide clear directions to the marker**, e.g. continued on page 20.
- SHOW ALL YOUR WORK, **clearly and in order**, if you wish to receive full credit.
- **No textbook, lecture note, calculator, computer, or other aid of any sort is allowed**.
- PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer the exam questions as written.
- Good luck!

Q1	Q2	Q3	Q4	Q5	Q6	Total
20	10	15	20	20	15	100

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Problem 1

Consider the real 3×3 matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) **Determine the set of all eigenvalues of** A. [5 pts]

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(Problem 1 - Cont'd)

(b) **Determine whether or not A is invertible**. [5 pts]

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(Problem 1 - Cont'd)

(c) **Determine whether or not A is diagonalizable**. [10 pts]

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Problem 2

Let A be a real 2×2 matrix, and assume A has eigenvalues $\lambda_1, \lambda_2 \in \mathbb{R}$ with $\lambda_1 \neq \lambda_2$. **Determine whether or not A is diagonalizable**. [10 pts]

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Problem 3

Let $(\mathbf{V}, +, \cdot)$ be a real vector space.

(a) Assume first $\mathbf{V}$ has dimension 3, and let $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ be a basis for $\mathbf{V}$. Let $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3 \in \mathbf{V}$ be defined by $\mathbf{w}_1 = \mathbf{v}_1$, $\mathbf{w}_2 = \mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{w}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$. **Determine whether or not $(\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3)$ is a basis for $\mathbf{V}$.** [5 pts]

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(Problem 3 - Cont'd)

(b) Let now $\mathbf{v}_1, \mathbf{v}_2$ be linearly independent vectors of $\mathbf{V}$, and let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \in \mathbf{V}$ be defined by $\mathbf{u}_1 = \mathbf{v}_1, \mathbf{u}_2 = \mathbf{v}_1 - \mathbf{v}_2, \mathbf{u}_3 = \mathbf{v}_1 + \mathbf{v}_2$. Let $\mathbf{U}$ denote the linear span of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$. **Compute the dimension of $\mathbf{U}$**. `[5 pts]`

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(Problem 3 - Cont'd)

(c) Consider now the real vector space $\widehat{\mathbb{R}^3}$, and let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \widehat{\mathbb{R}^3}$ be defined by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ k \\ k^2 \end{pmatrix},$$

where $k \in \mathbb{R}$. **Determine for which values of k we have that $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ forms a basis for $\widehat{\mathbb{R}^3}$.** [5 pts]

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Problem 4

Consider the system of linear equations given by:

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 1, \\ x_1 + 2x_2 + 3x_3 &= 4, \\ ax_1 + ax_2 + 2ax_3 + x_4 &= 5, \end{aligned}$$

where we wish to solve for the quadruple (x_1, x_2, x_3, x_4) of real numbers, and where a is a real parameter.

(a) **Write the augmented matrix for this system.** `[5 pts]`

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(Problem 4 - Cont'd)

(b) **Transform the augmented matrix to row-echelon form using a sequence of elementary row operations (clearly indicate which elementary row operation you perform at each step).** [5 pts]

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(Problem 4 - Cont'd)

(c) **Using (b), determine all the values of a for which the system has no solution**. `[5 pts]`

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(Problem 4 - Cont'd)

(d) **Let now** $a = 5$**; determine the set of all solutions of the system using back-substitution**.
`[5 pts]`

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Problem 5

Consider the real 3×3 matrices A and B given by

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & a+1 & 0 \\ 1 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix},$$

where a is a real parameter.

(a) **Compute AB and BA.** `[5 pts]`

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(Problem 5 - Cont'd)

(b) **Compute the determinant** $\det(A)$ **of** A **and determine all the values of** a **for which** A **is invertible**. [5 pts]

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(Problem 5 - Cont'd)

(c) **Determine whether or not B is invertible by computing the determinant $\det(B)$ of B.** [5 pts]

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(Problem 5 - Cont'd)

(d) **Compute** $\det(AABBAB)$. `[5 pts]`

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Problem 6

Let A be the real 3×4 matrix given by

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 3 & 0 & 6 & -3 \\ -2 & 0 & -4 & 2 \end{pmatrix}.$$

(a) **Find a basis for** $\mathrm{Ker}(A)$ **and compute the dimension of** $\mathrm{Ker}(A)$. `[5 pts]`

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(Problem 6 - Cont'd)

(b) **Find a basis for** $\mathrm{Im}(A)$ **and compute the dimension of** $\mathrm{Im}(A)$. `[5 pts]`

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(Problem 6 - Cont'd)

(c) **Verify the rank-nullity theorem using the dimensions computed in (a) and (b).** `[5 pts]`

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Space for additional work. **Indicate clearly which question you are continuing if you use this space.**

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