STUDENT NUMBER: ______________________________

# APSC 174 — Midterm 1

Monday February 11, 2019

T. Linder M. Roth S. Yüksel

First name (please write as legibly as possible within the boxes)

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | |

Last name

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | |

Student ID number

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | |

Instructions: The exam has six questions, worth a total of 100 marks.

Answer **all questions**, writing clearly in the space provided. If you need more room, continue to answer on the back of the **previous page**, providing clear directions on where to find the continuation of your answer.

To receive full credit you must show your work, clearly and in order.

No textbook, lecture notes, calculator, computer, or other aid, is allowed.

Good luck!

| **1** | **2** | **3** | **4** | **5** | **6** | Total |
|---|---|---|---|---|---|---|
| /15 | /20 | /15 | /15 | /20 | /15 | /100 |

STUDENT NUMBER:

**1.** Consider the set

$$\mathbf{V} = \{(x, y) \ : \ x, y \in \mathbb{R},\ y > 0\}$$

with the addition and scalar multiplication rules given by

**Addition:** For any $(x_1, y_1)$, $(x_2, y_2) \in \mathbf{V}$,

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 - 2,\ y_1 y_2).$$

**Scalar Multiplication:** For any $\alpha \in \mathbb{R}$, $(x, y) \in \mathbf{V}$,

$$\alpha \cdot (x, y) = (\alpha x - 2\alpha + 2,\ y^{\alpha}).$$

It can be proved (and you do not have to do this) that $\mathbf{V}$ with these operations is a vector space.

[5 pts] (a) Compute (using the operations on $\mathbf{V}$) the linear combination $2 \cdot (3, 5) + 5 \cdot (4, 1)$.

[5 pts] (b) Determine the zero vector $\mathbf{0}$ of $\mathbf{V}$.

STUDENT NUMBER: ______

[5 pts] (c) For $(x, y) \in \mathbf{V}$, determine its additive inverse; that is, find a vector $(x', y')$ such that $(x, y) + (x', y') = \mathbf{0}$.

STUDENT NUMBER:

**2.** Let

$$\mathbf{W} = \Big\{(x, y, z) \in \mathbb{R}^3 \ : \ 3x - 2y + z = 0\Big\} \subset \mathbb{R}^3.$$

Here the operations on $\mathbb{R}^3$ are the usual ones of addition and scalar multiplication.

[10 pts] (a) Show, with proof, that $\mathbf{W}$ is a subspace of $\mathbb{R}^3$.

[10 pts] (b) Let $\mathbf{v}_1 = (1, 0, -3)$ and $\mathbf{v}_2 = (0, 1, 2)$. Show that $S_{(\mathbf{v}_1, \mathbf{v}_2)} \subset \mathbf{W}$.

STUDENT NUMBER:

---

**3.** Recall that $C^\infty(\mathbb{R})$ is the vector space of functions from $\mathbb{R}$ to $\mathbb{R}$ which have a first, second, third, $\ldots$, etc, derivative. The operations on $C^\infty(\mathbb{R})$ are addition and scalar multiplication of functions as used in class. Let

$$\mathbf{W} = \left\{ f \in C^\infty(\mathbb{R}) \ : \ \int_{-\pi}^{\pi} f(x)\,dx = 0 \right\} \subset C^\infty(\mathbb{R}).$$

For instance, $x \in \mathbf{W}$ since

$$\int_{-\pi}^{\pi} x\,dx = \left.\frac{x^2}{2}\right|_{x=-\pi}^{x=\pi} = \frac{1}{2}\left((\pi)^2 - (-\pi)^2\right) = 0,$$

while $x^2 \notin \mathbf{W}$ since $\int_{-\pi}^{\pi} x^2\,dx = \frac{2}{3}\pi^3 \neq 0$.

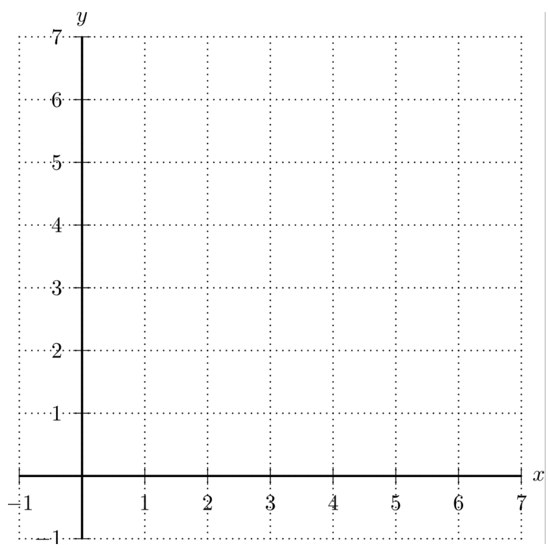
[5 pts] (a) Is $\sin(x) \in \mathbf{W}$?

[10 pts] (b) Determine, with proof, whether or not $\mathbf{W}$ is a subspace of $C^\infty(\mathbb{R})$.

STUDENT NUMBER:

**4.**

[5 pts] (a) In the grid below, draw and label the vector $(2, 4)$ and the vectors $(0, 6)$, $(2, 6)$, and $(4, 6)$.



STUDENT NUMBER: ______

[10 pts] (b) For which values of $t \in \mathbb{R}$ do the vectors $(2, 4)$ and $(t, 6)$ span $\mathbb{R}^2$?

STUDENT NUMBER:

**5.**

[5 pts] (a) State what it means for a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ to be *linearly independent*.

[5 pts] (b) State what it means for a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ to be *linearly dependent*.

[10 pts] (c) Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, are elements of a vector space $\mathbf{V}$, and we know that the set $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ is linearly *independent*, and also that the set $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ is linearly *dependent*. Show that $\mathbf{v}_3 \in S_{(\mathbf{v}_1,\mathbf{v}_2)}$.

STUDENT NUMBER:

**6.** Let $\mathbf{v}_1 = (4, 3, 1)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (0, 1, -1)$, and $\mathbf{v}_4 = (2, 0, 3)$ in $\mathbb{R}^3$.

[5 pts] (a) Is $\mathbf{v}_1$ a linear combination of $\mathbf{v}_2$ and $\mathbf{v}_3$?

[5 pts] (b) Determine, with proof, whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent or linearly independent.

[5 pts] (c) Determine, with proof, whether the set $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent or linearly independent.