STUDENT NUMBER:

APSC 174 — Midterm 1

Monday February 10, 2020

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Instructions: The exam has **five** questions, worth a total of 100 marks.

Answer **all questions**, writing clearly in the space provided, including the provided space for additional work. If you need more room, continue to answer on the back of the **previous page**, providing clear directions on where to find the continuation of your answer.

To receive full credit you must show your work, clearly and in order.

No textbook, lecture notes, calculator, computer, or other aid, is allowed.

Good luck!

1	**2**	**3**	**4**	**5**	Total
/20	/20	/15	/20	/25	/100

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1. In the vector space $\mathbb{R}^3 = \{(x, y, z) \ : \ x, y, z \in \mathbb{R}\}$ under the usual addition and scalar multiplication operations seen in class, consider the vectors $\mathbf{w} = (3, 6, 7)$, $\mathbf{v}_1 = (1, 2, 0)$, $\mathbf{v}_2 = (1, 1, 1)$, and $\mathbf{v}_3 = (0, 0, 1)$.

[8 pts] (a) Is $\mathbf{w}$ a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$? (Justify your answer.)

[6 pts] (b) Is the set $\{\mathbf{w}, \mathbf{v}_1, \mathbf{v}_2\}$ linearly dependent, or linearly independent? (Justify your answer.)

[6 pts] (c) Is the set $\{\mathbf{w}, \mathbf{v}_1, \mathbf{v}_3\}$ linearly dependent, or linearly independent? (Justify your answer.)

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2. Consider the vector space $\mathbb{R}^2 = \{(x, y) \ : \ x, y \in \mathbb{R}\}$ under the usual addition and scalar multiplication operations seen in class.

[6 pts] (a) Determine, with proof, whether or not the set $\mathbf{H}_1 = \{(x, y) \in \mathbb{R}^2 \ : \ y \geq x\}$ is a vector subspace of $\mathbb{R}^2$.

[6 pts] (b) Given the set $\mathbf{H}_2 = \{(x, y) \in \mathbb{R}^2 \ : \ y \leq x\}$, determine the set $\mathbf{H}_1 \cap \mathbf{H}_2$.

[8 pts] (c) Determine, with proof, whether or not $\mathbf{H}_1 \cap \mathbf{H}_2$ is a vector subspace of $\mathbb{R}^2$.

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3. Recall that $C^\infty(\mathbb{R})$ is the vector space of functions from $\mathbb{R}$ to $\mathbb{R}$ that can be differentiated arbitrarily many times. The operations on $C^\infty(\mathbb{R})$ are the usual addition and scalar multiplication of functions as seen in class. Let

$$\mathbf{W} = \left\{ f \in C^\infty(\mathbb{R}) \; : \; f''(0) + f'(\pi/2) = 0 \right\} \subset C^\infty(\mathbb{R}).$$

For instance, $\sin(x) \in \mathbf{W}$ since $\sin(x)' = \cos(x)$, $\sin(x)'' = -\sin(x)$, and so

$$\sin(0)'' + \sin(\pi/2)' = -\sin(0) + \cos(\pi/2) = -0 + 0 = 0.$$

[5 pts] (a) Is $\cos(x) \in \mathbf{W}$?

[10 pts] (b) Determine, with proof, whether or not $\mathbf{W}$ is a subspace of $C^\infty(\mathbb{R})$.

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4. Consider a vector space $\mathbf{V}$. Answer the questions below about this vector space.

[5 pts] (a) Define what it means for a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ in $\mathbf{V}$ to be *linearly dependent*.

[5 pts] (b) Let $\mathbf{0}$ be the zero vector in $\mathbf{V}$ and let $\mathbf{v} \in \mathbf{V}$ be an arbitrary non-zero vector. Determine, with proof, whether the set $\{\mathbf{0}, \mathbf{v}\}$ is linearly dependent or independent.

[5 pts] (c) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a set of vectors in $\mathbf{V}$. Define what it means for a vector $\mathbf{v} \in \mathbf{V}$ to be in the span $S_{(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)}$.

[5 pts] (d) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a set of vectors in $\mathbf{V}$ and assume that $\mathbf{v} \in S_{(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)}$. Is the set $\{\mathbf{v}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ linearly dependent or independent? Justify (i.e., provide an argument for, or prove) your answer.

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5. Consider the set $\mathbf{V} = \{(x, y, z) \,:\, x, y, z \in \mathbb{R},\, z > 0\}$ with the following **new** addition and scalar multiplication operations:

Addition: For any (x_1, y_1, z_1), $(x_2, y_2, z_2) \in \mathbf{V}$,

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2,\; y_1 + y_2 - 3,\; z_1 z_2).$$

Scalar Multiplication: For any $\alpha \in \mathbb{R}$, $(x, y, z) \in \mathbf{V}$,

$$\alpha \cdot (x, y, z) = (\alpha x,\; \alpha y - 3\alpha + 3,\; z^{\alpha}).$$

It can be proved (and you do not have to do this) that $\mathbf{V}$ with these operations is a vector space.

[5 pts] (a) Determine $2 \cdot \big((-5, -3, 2) + (6, 4, 3)\big)$ using the operations in $\mathbf{V}$.

[5 pts] (b) Determine the zero vector $\mathbf{0}$ of $\mathbf{V}$.

[5 pts] (c) Given the vector $(x, y, z) \in \mathbf{V}$, determine its additive inverse; that is, find a vector (x', y', z') such that $(x, y, z) + (x', y', z') = \mathbf{0}$.

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(— problem 5 continued —)

[5 pts] (d) Given $\mathbf{w}_1 = (0, 3, 1)$, $\mathbf{w}_2 = (-1, 2, 3)$ and $\mathbf{w}_3 = (2, 2, 9)$ in $\mathbf{V}$, determine (using the operations in $\mathbf{V}$) whether or not $\mathbf{w}_3$ is a linear combination of $\mathbf{w}_1$ and $\mathbf{w}_2$.

[5 pts] (e) Determine, with proof (using the operations in $\mathbf{V}$), whether or not the set of the above vectors $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is linearly independent.

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