STUDENT NUMBER: ______________________________

APSC 174 — Midterm 2

Monday March 25, 2019

T. Linder   M. Roth   S. Yüksel

INSTRUCTIONS: The exam has six questions, worth a total of 100 marks.

Answer **all questions**, writing clearly in the space provided. If you need more room, continue to answer on the back of the **previous page**, providing clear directions on where to find the continuation of your answer.

To receive full credit you must show your work, clearly and in order.

No textbook, lecture notes, calculator, computer, or other aid, is allowed.

Good luck!

1	**2**	**3**	**4**	**5**	**6**	Total
/15	/15	/20	/20	/15	/15	/100

STUDENT NUMBER:

[15 pts] **1.** Let $M = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 3 & -1 & 6 & 7 \\ 1 & 1 & 2 & 5 \end{bmatrix}$.

Use row operations to put M into row reduced echelon form (RREF). Show your steps.

STUDENT NUMBER:

[15 pts] **2.** Parametrize all the solutions to the system of linear equations corresponding to the following augmented matrix. Write your parametrization in vector form (x_1, x_2, $\ldots$, x_6 are good names for the variables).

$$\left[\begin{array}{cccccc|c} 1 & 0 & 4 & 0 & 2 & 0 & -1 \\ 0 & 1 & 6 & 0 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

STUDENT NUMBER:

3. Suppose that $L\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ is a linear transformation, and we know that $L(2,3) = (3,0,2)$ and that $L(3,4) = (2,1,1)$.

[5 pts] (a) Write $(1,0)$ and $(0,1)$ as linear combinations of $(2,3)$ and $(3,4)$.

[10 pts] (b) Find the standard matrix for L.

[5 pts] (c) Find $L(4,5)$.

STUDENT NUMBER:

4. Let $L\colon \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ be the linear transformation given by the rule

$$L(x, y, z, w) = (x + 2z + w,\ y + 4z + 3w,\ 2x - y - w).$$

[5 pts] (a) Find the standard matrix for L.

[5 pts] (b) Find a basis for $\mathrm{Im}(L)$.

Not covered in Midterm 2 (of 2021)

[5 pts] (c) Find $\dim(\mathrm{Im}(L))$.

[5 pts] (d) Find $\dim(\mathrm{Ker}(L))$.

STUDENT NUMBER:

5. Let $\mathbf{V}$ and $\mathbf{W}$ be vector spaces, $L\colon \mathbf{V} \longrightarrow \mathbf{W}$ a linear transformation, and $\mathbf{v}_1, \ldots, \mathbf{v}_p$ vectors in $\mathbf{V}$.

[5 pts] (a) Suppose that $\alpha_1, \ldots, \alpha_p \in \mathbb{R}$ are such that $\alpha_1 L(\mathbf{v}_1) + \alpha_2 L(\mathbf{v}_2) + \cdots + \alpha_p L(\mathbf{v}_p) = \mathbf{0}$.

Show that $\alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \cdots + \alpha_p \mathbf{v}_p \in \mathrm{Ker}(L)$.

[10 pts] (b) Suppose in addition that L is injective, and that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is linearly independent.

Prove that $\{L(\mathbf{v}_1), L(\mathbf{v}_2), \ldots, L(\mathbf{v}_p)\}$ is linearly independent.

STUDENT NUMBER:

---

6. Let $\mathcal{P}_2(\mathbb{R})$ be the vector space of polynomials of degree $\leqslant 2$, with real coefficients. That is, the polynomials of the form

$$a_0 + a_1 x + a_2 x^2$$

with $a_0,\ a_1,\ a_2 \in \mathbb{R}$. The operations on $\mathcal{P}_2(\mathbb{R})$ are the usual addition and scalar multiplication of polynomials.

[10 pts] (a) Find, with proof, a basis for $\mathcal{P}_2(\mathbb{R})$.

[5 pts] (b) What is the dimension of $\mathcal{P}_2(\mathbb{R})$?