STUDENT NUMBER:

APSC 174 – Midterm 2

March 2022

A. Ableson F. Alajaji I. Dimitrov K. Zhang

Instructions:

The exam has **five** questions, worth a total of 100 marks.

Answer **all 5 questions**, writing clearly in the space provided, including the provided space for additional work. If you need more room, continue to answer on the **next blank page**, providing clear directions on where to find the continuation of your answer.

To receive full credit you must show your work, clearly and in order.

Correct answers without adequate explanations will not receive full marks.

No textbook, lecture notes, calculator, or other aid, is allowed.

Good luck!

1	**2**	**3**	**4**	**5**	Total
/20	/20	/20	/20	/20	/100

STUDENT NUMBER:

1. Consider the linear system of equations shown below:

$$\begin{aligned} x + y + 2z &= 1 \\ -x + 3y + 6z &= -9 \\ 2y + 4z &= -4 \end{aligned}$$

[4 pts] (a) Write out the augmented matrix of the system above.

[12 pts] (b) Use row operations to reduce the matrix obtained in (a) to RREF.

[4 pts] (c) Write out the set of solutions for the original linear system, defining and using free variables as necessary.

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

2. Suppose that $L : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation (map) and we know that $L(1,4) = (-2,-3,2)$ and $L(2,3) = (1,-1,4)$.

[5 pts] (a) Write $(1,0)$ and $(0,1)$ as linear combinations of $(1,4)$ and $(2,3)$.

[5 pts] (b) Determine $L(1,0)$ and $L(0,1)$.

[5 pts] (c) Determine $L(2,-1)$.

[5 pts] (d) Determine the formula for $L(x,y)$ for any $(x,y) \in \mathbb{R}^2$.

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

3. Consider the vector space $P_2(\mathbb{R})$ of polynomial functions of degree at most 2; i.e., any member $f \in P_2(\mathbb{R})$ has the form $f(x) = a + bx + cx^2$ for some real values of a, b and c.

Now consider the following polynomial functions in $P_2(\mathbb{R})$:

$$p_1(x) = x^2$$
$$p_2(x) = x + 2x^2$$
$$p_3(x) = 1 + 2x + 3x^2$$

for $x \in \mathbb{R}$.

[7 pts] (a) Determine if the set $\{p_1, p_2, p_3\}$ is a generating set for $P_2(\mathbb{R})$.

[7 pts] (b) Determine if the set $\{p_1, p_2, p_3\}$ is linearly independent.

[6 pts] (c) Determine the dimension of $P_2(\mathbb{R})$.

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

4. Let $L : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation (map) defined by

$$L(x, y, z, t) = (x + 2y - t, 2x + 4y - 2t, x + 2y + z + t).$$

[4 pts] (a) Find the standard matrix of L.

[2 pts] (b) Determine whether the vector $(1, 1, -2, 1)$ belongs to $\mathrm{Ker}(L)$.

[4 pts] (c) Determine whether the vector $(0, 0, 4)$ belongs to $\mathrm{Im}(L)$.

[6 pts] (d) Find two linearly independent vectors in $\mathrm{Ker}(L)$.

[4 pts] (e) Decide, with proof, if L is injective (i.e., one-to-one).

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

5. Answer the following questions.

[10 pts] (a) Recall the weird vector space seen in class $\mathbf{W}_2 = \{(x, y) : x, y \in \mathbb{R},\, x, y > 0\}$ under the following addition and scalar multiplication operations, denoted by $\oplus$ and $\odot$, respectively:

Addition: For any (x_1, y_1), $(x_2, y_2) \in \mathbf{W}_2$, we have that $(x_1, y_1) \oplus (x_2, y_2) = (x_1 x_2,\, y_1 y_2)$.

Scalar Multiplication: For any $\alpha \in \mathbb{R}$, $(x, y) \in \mathbf{W}_2$, we have that $\alpha \odot (x, y) = (x^\alpha,\, y^\alpha)$.

Now consider the transformation $L : C^\infty(\mathbb{R}) \to \mathbf{W}_2$ be defined by

$$L(f) = \left(e^{f(0)}, e^{f(5)}\right), \qquad f \in C^\infty(\mathbb{R}).$$

Prove or disprove that L is a linear transformation.

[10 pts] (b) Let $\mathbf{V}$ and $\mathbf{W}$ be vector spaces and let $L : \mathbf{V} \to \mathbf{W}$ be a linear map. Let $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ be vectors in $\mathbf{V}$ such that the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent. Prove or disprove that the set $\{L(\mathbf{v}_1), L(\mathbf{v}_2), L(\mathbf{v}_3)\}$ is linearly dependent.

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]

STUDENT NUMBER:

[Space for additional work. Indicate clearly which question you are continuing if you use this page.]