Queen's University - APSC174 **Winter 2025**

Tutorial 11

1. Determine if the linear transformations described by the following matrices are invertible. If not, explain why, and if so, find the matrix of the inverse transformation.

(a) $\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 6 \\ 0 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 3 \\ 9 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & 6 & 1 & 0 \\ 7 & 10 & 4 & 1 \end{bmatrix}$

2. Suppose that A is the matrix

$$A = \begin{bmatrix} 5 & 2 & 4 \\ 2 & 3 & 1 \\ 5 & 6 & 3 \end{bmatrix}.$$

(a) Find the inverse of A.

(b) Explain why, for any values of a, b, and c, the equations

$$\begin{array}{rcrcrcl} 5x & + & 2y & + & 4z & = & a \\ 2x & + & 3y & + & z & = & b \\ 5x & + & 6y & + & 3z & = & c \end{array}$$

always have a unique solution.

(c) Find this unique solution (in terms of a, b, and c).

3. Find the inverse of the following matrix:

$$A = \begin{bmatrix} 7 & 14 & -6 \\ 1 & 2 & -1 \\ 3 & 7 & -3 \end{bmatrix}$$

4. Find the determinant of each of the following matrices.

(a) $A = \begin{bmatrix} 7 & 4 & 6 & 2 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 7 & -3 \\ 0 & 0 & 0 & -6 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & -1 \\ 3 & 2 & 3 \end{bmatrix}$ (c) $C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 9 \\ 2 & 0 & 6 \end{bmatrix}$

5. Let $a \in \mathbb{R}$ be a real number and consider the following 3×3 real matrices:

$$A = \begin{bmatrix} 1 & 1 & a \\ -1 & a & 1 \\ a & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & -8 \\ 3 & 2 & -2 \\ 1 & 1 & 1 \end{bmatrix},$$

(a) Compute the determinant $\det(B)$ of B, and use it to determine whether or not B is invertible.

(b) Compute the determinant $\det(A)$ of A, and use it to determine all the values of a for which A is invertible.

(c) Let C be the matrix product $C = B^2$ (recall that $B^2 = BB$). Compute C and determine whether or not C is invertible.