Tutorial 01 Questions

1. Translate the following problem into a system of equations and then solve. A river cruise ship sailed 60 kilometers downstream for 4 hours and then took 5 hours sailing upstream to return to the dock. Find the speed of the ship in still water and the speed of the river current.

2. One can often describe a given set in different ways. For instance, the sets $\{x^2 : x \in R\}$ and $\{x \in R : x \geq 0\}$ are the same set. Each of the six sets given below is equal to one of the others. Match up each set with the other set on the list it is equal to.

   (a) $\{x^2 : x \in Z\}$
   (b) $\{\cos^2(x) + 1 : x \in R\}$
   (c) $Z \cap \{x \in R : -\frac{1}{2} \leq x \leq \frac{9}{2}\}$
   (d) $\{x - \frac{1}{2} : x \in Z\} \cup Z$
   (e) $\{|x| : x \in Z, -4 \leq x \leq 4\}$
   (f) $\{2(x-1) : \frac{3}{2} \leq x \leq 2\}$

3. Let A and B be sets. Prove that if $A \subset B$ then $A \cap B = A$.

4. For each of the cases below, determine whether the given rule defines a valid function between the domain and target set given. If it does, also determine whether the function is injective, surjective, bijective, or none of these.

   (a) $f : N \to N,\ f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ \frac{x-1}{2} & \text{if } x \text{ is odd} \end{cases}$
   (b) $f : Z \to Z,\ f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$
   (c) $g : Z \to Z,\ g(x) = \sin\left(\frac{\pi x}{2}\right)$
   (d) $g : Z \to Z,\ g(x) = x^2 \cos\left(\frac{\pi x}{2}\right)$
   (e) $g : R \to R^2,\ g(t) = (t^2, t^3)$
   (f) $h : Q^+ \to N,\ h\left(\frac{m}{n}\right) = 2^m 3^n$, where $Q^+ = \left\{\frac{m}{n} : m, n \in N, m > 0, n > 0\right\}$