Tutorial 01 Solutions

1. **Solution.** Let x be the rate of the ship in still water and y be the rate of the current. Since rate times time is distance, we can write the system of equations:

$$4(x+y) = 60, \quad 5(x-y) = 60.$$

Solve the system:

$$20(x+y) = 300, \quad 20(x-y) = 240.$$

Adding these gives $40x = 540$, so $x = 13.5$. Substituting $x = 13.5$ back, we find $y = 1.5$. The rate of the ship is $x = 13.5$ km/h, and the rate of the current is $y = 1.5$ km/h.

2. **Solution.** To match up the sets, we evaluate each set:

   (a) $\{x^2 : x \in Z\}$ is the set of integers divided by 2.

   (b) $\{\cos^2(x) + 1 : x \in R\} = [1, 2]$.

   (c) $Z \cap \{x \in R : -\frac{1}{2} \leq x \leq \frac{9}{2}\} = \{0, 1, 2, 3, 4\}$.

   (d) $\{x - \frac{1}{2} : x \in Z\} \cup Z$ is the set of integers and half-integers.

   (e) $\{|x| : x \in Z, -4 \leq x \leq 4\} = \{0, 1, 2, 3, 4\}$.

   (f) $\{2(x-1) : \frac{3}{2} \leq x \leq 2\} = [1, 2]$.

   Thus, the sets match as follows: (a) $\leftrightarrow$ (d), (b) $\leftrightarrow$ (f), (c) $\leftrightarrow$ (e).

3. **Solution.** To prove $A \cap B = A$ when $A \subset B$:

   (a) By definition of intersection, $A \cap B \subset A$.

   (b) Since $A \subset B$, any $x \in A$ is also in B, so $A \subset A \cap B$.

   Combining, $A \cap B = A$.

4. **Solution.** For each case:

   (a) $f : N \to N$ defines a surjective but not injective function.

   (b) $f : Z \to Z$ defines a bijective function.

   (c) $g : Z \to Z$ is neither injective nor surjective.

(d) $g : Z \to Z$ is surjective but not injective.

(e) $g : R \to R^2$ is injective but not surjective.

(f) $h : Q^+ \to N$ does not define a function due to non-uniqueness.