Queen's University - APSC174 **Winter 2025**

Tutorial 03

1. Consider the usual vector space $(\mathbb{R}^2, +, \cdot)$. For each of the following subsets of $\mathbb{R}^2$, determine whether or not it is a vector subspace of $\mathbb{R}^2$:

(a) S = set of all (x, y) in $\mathbb{R}^2$ such that $6x + 8y = 0$.

(b) S = set of all (x, y) in $\mathbb{R}^2$ such that $6x + 8y = 1$.

2. Recall that $\mathcal{F}(\mathbb{R};\mathbb{R})$ denotes the vector space of functions $f : \mathbb{R} \to \mathbb{R}$ with the usual function addition and scalar multiplication. For each of the following subsets of $\mathcal{F}(\mathbb{R};\mathbb{R})$, determine whether or not it is a vector subspace of $\mathcal{F}(\mathbb{R};\mathbb{R})$:

(a) S = set of all f in $\mathcal{F}(\mathbb{R};\mathbb{R})$ such that $f(x) + f(x+1) + f(x+2) = 1$ for all $x \in \mathbb{R}$.

(b) S = set of all f in $\mathcal{F}(\mathbb{R};\mathbb{R})$ such that $f(x) + f(x+1) + f(x+2) = 0$ for all $x \in \mathbb{R}$.

3. Consider the following vector space

$$\mathbf{W}_2 = \{(x_1, x_2) : \ x_1, x_2 \in \mathbb{R},\ x_1 > 0,\ x_2 > 0\}$$

under the following operations:

Addition: for any $\mathbf{u} = (x_1, x_2)$ and $\mathbf{v} = (y_1, y_2)$ in $\mathbf{W}_2$,

$$\mathbf{u} + \mathbf{v} = (x_1, x_2) + (y_1, y_2) = (x_1 y_1, x_2 y_2)$$

Scalar multiplication: for any $\alpha \in \mathbb{R}$ and $\mathbf{u} = (x_1, x_2) \in \mathbf{W}_2$,

$$\alpha \cdot \mathbf{u} = \alpha \cdot (x_1, x_2) = (x_1^{\alpha}, x_2^{\alpha}) \,.$$

(a) Determine which of the following subsets of $\mathbf{W}_2$ is a subspace of $\mathbf{W}_2$:

(a1) S = set of all (x_1, x_2) in $\mathbf{W}_2$ such that $x_1 x_2 = 0$.

(a2) S = set of all (x_1, x_2) in $\mathbf{W}_2$ such that $x_1^2 x_2 = 1$.

(b) Consider the following vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ in $\mathbf{W}_2$ defined as follows:

$$\mathbf{v}_1 = (1, 2), \quad \mathbf{v}_2 = (2, 1), \quad \mathbf{v}_3 = (3, 2).$$

(b1) Is $\mathbf{v}_1$ in the linear span of $\{\mathbf{v}_2\}$?

(b2) Is $\mathbf{v}_3$ in the linear span of $\{\mathbf{v}_1, \mathbf{v}_2\}$?

4. Consider $\mathbb{R}^3$ with the usual (component-wise) addition and scalar multiplication operations. Show that the linear span of the vectors $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 1)$ is $\mathbb{R}^3$ itself.