1. Let $L\colon \mathbf{V} \longrightarrow \mathbf{W}$ be a linear transformation, with $\mathbf{V}$ and $\mathbf{W}$ finite dimensional vector spaces.

   (a) Prove that L is surjective if and only if $\dim(\mathrm{Im}(L)) = \dim(\mathbf{W})$. NOTES: (1) Since the question is an "if and only if", there are two directions to prove; make sure you understand what each of the directions is. In one of the directions you will need the result of **Tutorial 7**, **5(b)**.

   (b) Show that L is injective if and only if $\dim(\mathrm{Ker}(L)) = 0$.

A linear transformation $L\colon \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is given in each of Problems 2–5 below. For each of them :

(a) Find the standard matrix for L.

(b) Find a basis for $\mathrm{Im}(L)$.

(c) Find a basis for $\mathrm{Ker}(L)$.

(d) Find $\dim(\mathrm{Im}(L))$.

(e) Find $\dim(\mathrm{Ker}(L))$.

(f) What does the Rank-Nullity theorem predict for L? Does it hold in this case?

(g) Is L injective?

(h) Is L surjective?

For parts (g) and (h) you should use the results of Problem 1.

2. $L_1\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ is the linear transformation given by

$$L_1(x, y, z) = (2x + y - z,\ -x + 2y - 7z,\ 3y - 9z,\ -4x + y - 7z).$$

3. $L_2\colon \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ is the linear transformation given by

$$L_2(x, y, z, w) = (2x - y + 4z + w,\ x + 3y + 9z + 2w,\ 2y + 4z + 2w).$$

4. $L_3\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is the linear transformation given by

$$L_3(x, y, z) = (3x + 2y + 2z,\ -x + 3y + 3z,\ y + z).$$

5. $L_4\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is the linear transformation given by

$$L_4(x, y) = (2x + 3y,\ x + y).$$